Question 12.1:

Choose the correct alternative from the clues given at the end of the each statement:

The size of the atom in Thomson's model is .... the atomic size in Rutherford's model. (much greater than/no different from/much less than.)

In the ground state of .... electrons are in stable equilibrium, while in .... electrons always experience a net force.

(Thomson's model/ Rutherford's model.)

A *classical* atom based on .... is doomed to collapse.

(Thomson's model/ Rutherford's model.)

An atom has a nearly continuous mass distribution in a .... but has a highly non-uniform mass distribution in ....

(Thomson's model/ Rutherford's model.)

The positively charged part of the atom possesses most of the mass in .... (Rutherford's model/both the models.)

Answer

The sizes of the atoms taken in Thomson's model and Rutherford's model have the same order of magnitude.

In the ground state of Thomson's model, the electrons are in stable equilibrium. However, in Rutherford's model, the electrons always experience a net force.

A *classical* atom based on Rutherford's model is doomed to collapse.

An atom has a nearly continuous mass distribution in Thomson's model, but has a highly non-uniform mass distribution in Rutherford's model.

The positively charged part of the atom possesses most of the mass in both the models.

Question 12.2:

Suppose you are given a chance to repeat the alpha-particle scattering experiment using a thin sheet of solid hydrogen in place of the gold foil. (Hydrogen is a solid at temperatures below 14 K.) What results do you expect?

Answer

In the alpha-particle scattering experiment, if a thin sheet of solid hydrogen is used in place of a gold foil, then the scattering angle would not be large enough. This is because the mass of hydrogen (1.67×10^{-27} kg) is less than the mass of incident α−particles (6.64×10^{-27} kg). Thus, the mass of the scattering particle is more than the target nucleus (hydrogen). As a result, the α−particles would not bounce back if solid hydrogen is used in the α-particle scattering experiment.

Question 12.3:

What is the shortest wavelength present in the Paschen series of spectral lines?

Answer

Rydberg's formula is given as:

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19} \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$$

Where,

h = Planck's constant = 6.6×10^{-34} Js

c = Speed of light = 3×10^8 m/s

(n_1 and n_2 are integers)

The shortest wavelength present in the Paschen series of the spectral lines is given for values $n_1 = 3$ and $n_2 = \infty$.

$$\frac{hc}{\lambda} = 21.76 \times 10^{-19}\left[\frac{1}{(3)^2} - \frac{1}{(\infty)^2}\right]$$

$$\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8} \times 9}{21.76 \times 10^{-19}}$$

$$= 8.189 \times 10^{-7}\ \text{m}$$

$$= 818.9\ \text{nm}$$

Question 12.4:

A difference of 2.3 eV separates two energy levels in an atom. What is the frequency of radiation emitted when the atom makes a transition from the upper level to the lower level?

Answer

Separation of two energy levels in an atom,

E = 2.3 eV

= 2.3 × 1.6 × 10^{-19}

= 3.68 × 10^{-19} J

Let ν be the frequency of radiation emitted when the atom transits from the upper level to the lower level.

We have the relation for energy as:

$E = h\nu$

Where,

h = Planck's constant $= 6.62 \times 10^{-34}$ Js

$$\therefore \nu = \frac{E}{h}$$

$$= \frac{3.68 \times 10^{-19}}{6.62 \times 10^{-32}} = 5.55 \times 10^{14}\ \text{Hz}$$

Hence, the frequency of the radiation is 5.6×10^{14} Hz.

Question 12.5:

The ground state energy of hydrogen atom is −13.6 eV. What are the kinetic and potential energies of the electron in this state?

Answer

Ground state energy of hydrogen atom, $E = -13.6$ eV

This is the total energy of a hydrogen atom. Kinetic energy is equal to the negative of the total energy.

Kinetic energy $= -E = -(-13.6) = 13.6$ eV

Potential energy is equal to the negative of two times of kinetic energy.

Potential energy $= -2 \times (13.6) = -27.2$ eV

Question 12.6:

A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of the photon.

Answer

For ground level, $n_1 = 1$

Let E_1 be the energy of this level. It is known that E_1 is related with n_1 as:

$$E_1 = \frac{-13.6}{n_1^2} \text{ eV}$$
$$= \frac{-13.6}{1^2} = -13.6 \text{ eV}$$

The atom is excited to a higher level, $n_2 = 4$.

Let E_2 be the energy of this level.

$$\therefore E_2 = \frac{-13.6}{n_2^2} \text{ eV}$$
$$= \frac{-13.6}{4^2} = -\frac{13.6}{16} \text{ eV}$$

The amount of energy absorbed by the photon is given as:

$E = E_2 - E_1$

$$= \frac{-13.6}{16} - \left(-\frac{13.6}{1}\right)$$
$$= \frac{13.6 \times 15}{16} \text{ eV}$$
$$= \frac{13.6 \times 15}{16} \times 1.6 \times 10^{-19} = 2.04 \times 10^{-18} \text{ J}$$

For a photon of wavelengthλ, the expression of energy is written as:

$$E = \frac{hc}{\lambda}$$

Where,

h = Planck's constant = 6.6×10^{-34} Js

c = Speed of light = 3×10^8 m/s

$$\therefore \lambda = \frac{hc}{E}$$
$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.04 \times 10^{-18}}$$
$$= 9.7 \times 10^{-8} \text{ m} = 97 \text{ nm}$$

And, frequency of a photon is given by the relation,

$$\nu = \frac{c}{\lambda}$$

$$= \frac{3\times10^{8}}{9.7\times10^{-8}} \approx 3.1 \times 10^{15} \text{ Hz}$$

Hence, the wavelength of the photon is 97 nm while the frequency is 3.1×10^{15} Hz.

Question 12.7:

Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the n = 1, 2, and 3 levels. (b) Calculate the orbital period in each of these levels.

Answer

Let v_1 be the orbital speed of the electron in a hydrogen atom in the ground state level, n_1 = 1. For charge (e) of an electron, v_1 is given by the relation,

$$v_1 = \frac{e^2}{n_1 4\pi \in_0 \left(h/2\pi\right)} = \frac{e^2}{2 \in_0 h}$$

Where,

$e = 1.6 \times 10^{-19}$ C

$\in_0$ = Permittivity of free space = 8.85×10^{-12} N^{-1} C^2 m^{-2}

h = Planck's constant = 6.62×10^{-34} Js

$$\therefore v_1 = \frac{\left(1.6\times10^{-19}\right)^2}{2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$= 0.0218 \times 10^{8} = 2.18 \times 10^{6}\ m/s$$

For level n_2 = 2, we can write the relation for the corresponding orbital speed as:

$$v_2 = \frac{e^2}{n_2 2 \in_0 h}$$

$$= \frac{\left(1.6\times10^{-19}\right)^2}{2\times2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$= 1.09 \times 10^6 \ m/s$$

And, for $n_3 = 3$, we can write the relation for the corresponding orbital speed as:

$$v_3 = \frac{e^2}{n_3 2 \in_0 h}$$

$$= \frac{\left(1.6\times10^{-19}\right)^2}{3\times2\times8.85\times10^{-12}\times6.62\times10^{-34}}$$

$$= 7.27 \times 10^5 \ m/s$$

Hence, the speed of the electron in a hydrogen atom in $n = 1$, n=2, and n=3 is 2.18×10^6 m/s, 1.09×10^6 m/s, 7.27×10^5 m/s respectively.

Let T_1 be the orbital period of the electron when it is in level $n_1 = 1$.

Orbital period is related to orbital speed as:

$$T_1 = \frac{2\pi r_1}{v_1}$$

Where,

r_1 = Radius of the orbit

$$= \frac{n_1^2 h^2 \in_0}{\pi m e^2}$$

h = Planck's constant = 6.62×10^{-34} Js

e = Charge on an electron = 1.6×10^{-19} C

$\in_0$ = Permittivity of free space = 8.85×10^{-12} $N^{-1} C^2 m^{-2}$

m = Mass of an electron = 9.1×10^{-31} kg

$$\therefore T_1 = \frac{2\pi r_1}{v_1}$$

$$= \frac{2\pi \times (1)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{2.18 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2}$$

$$= 15.27 \times 10^{-17} = 1.527 \times 10^{-16}\ s$$

For level $n_2 = 2$, we can write the period as:

$$T_2 = \frac{2\pi r_2}{v_2}$$

Where,

r_2 = Radius of the electron in $n_2 = 2$

$$= \frac{(n_2)^2 h^2 \in_0}{\pi m e^2}$$

$$\therefore T_2 = \frac{2\pi r_2}{v_2}$$

$$= \frac{2\pi \times (2)^2 \times \left(6.62 \times 10^{-34}\right)^2 \times 8.85 \times 10^{-12}}{1.09 \times 10^6 \times \pi \times 9.1 \times 10^{-31} \times \left(1.6 \times 10^{-19}\right)^2}$$

$$= 1.22 \times 10^{-15}\ s$$

And, for level $n_3 = 3$, we can write the period as:

$$T_3 = \frac{2\pi r_3}{v_3}$$

Where,

r_3 = Radius of the electron in $n_3 = 3$

$$= \frac{(n_3)^2 h^2 \in_0}{\pi m e^2}$$

$$\therefore T_3 = \frac{2\pi r_3}{v_3}$$

$$= \frac{2\pi \times (3)^2 \times (6.62 \times 10^{-34})^2 \times 8.85 \times 10^{-12}}{7.27 \times 10^5 \times \pi \times 9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2}$$

$$= 4.12 \times 10^{-15}\ s$$

Hence, the orbital period in each of these levels is 1.52×10^{-16} s, 1.22×10^{-15} s, and 4.12×10^{-15} s respectively.

Question 12.8:

The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. What are the radii of the $n = 2$ and $n = 3$ orbits?

Answer

The radius of the innermost orbit of a hydrogen atom, $r_1 = 5.3 \times 10^{-11}$ m.

Let r_2 be the radius of the orbit at $n = 2$. It is related to the radius of the innermost orbit as:

$$r_2 = (n)^2 r_1$$

$$= 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10}\ m$$

For $n = 3$, we can write the corresponding electron radius as:

$$r_3 = (n)^2 r_1$$

$$= 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10}\ m$$

Hence, the radii of an electron for $n = 2$ and $n = 3$ orbits are 2.12×10^{-10} m and 4.77×10^{-10} m respectively.

Question 12.9:

A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelengths will be emitted?

Answer

It is given that the energy of the electron beam used to bombard gaseous hydrogen at room temperature is 12.5 eV. Also, the energy of the gaseous hydrogen in its ground state at room temperature is −13.6 eV.

When gaseous hydrogen is bombarded with an electron beam, the energy of the gaseous hydrogen becomes −13.6 + 12.5 eV i.e., −1.1 eV.

Orbital energy is related to orbit level (*n*) as:

$$E = \frac{-13.6}{(n)^2} \text{ eV}$$

For $n = 3$, $E = \frac{-13.6}{9} = -1.5 \text{ eV}$

This energy is approximately equal to the energy of gaseous hydrogen. It can be concluded that the electron has jumped from $n = 1$ to $n = 3$ level.

During its de-excitation, the electrons can jump from $n = 3$ to $n = 1$ directly, which forms a line of the Lyman series of the hydrogen spectrum.

We have the relation for wave number for Lyman series as:

$$\frac{1}{\lambda} = R_y\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

Where,

R_y = Rydberg constant = $1.097 \times 10^7 \text{ m}^{-1}$

λ= Wavelength of radiation emitted by the transition of the electron

For $n = 3$, we can obtain λas:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{3^2} \right)$$
$$= 1.097 \times 10^7 \left(1 - \frac{1}{9} \right) = 1.097 \times 10^7 \times \frac{8}{9}$$
$$\lambda = \frac{9}{8 \times 1.097 \times 10^7} = 102.55 \; nm$$

If the electron jumps from $n = 2$ to $n = 1$, then the wavelength of the radiation is given as:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$$
$$= 1.097 \times 10^7 \left(1 - \frac{1}{4} \right) = 1.097 \times 10^7 \times \frac{3}{4}$$
$$\lambda = \frac{4}{1.097 \times 10^7 \times 3} = 121.54 \; nm$$

If the transition takes place from n = 3 to n = 2, then the wavelength of the radiation is given as:

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{2^2} - \frac{1}{3^2} \right)$$
$$= 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{9} \right) = 1.097 \times 10^7 \times \frac{5}{36}$$
$$\lambda = \frac{36}{5 \times 1.097 \times 10^7} = 656.33 \; nm$$

This radiation corresponds to the Balmer series of the hydrogen spectrum.

Hence, in Lyman series, two wavelengths i.e., 102.5 nm and 121.5 nm are emitted. And in the Balmer series, one wavelength i.e., 656.33 nm is emitted.

Question 12.10:

In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 m/s. (Mass of earth = 6.0×10^{24} kg.)

Answer

Radius of the orbit of the Earth around the Sun, $r = 1.5 \times 10^{11}$ m

Orbital speed of the Earth, $v = 3 \times 10^4$ m/s

Mass of the Earth, $m = 6.0 \times 10^{24}$ kg

According to Bohr's model, angular momentum is quantized and given as:

$$mvr = \frac{nh}{2\pi}$$

Where,

h = Planck's constant = 6.62×10^{-34} Js

n = Quantum number

$$\therefore n = \frac{mvr2\pi}{h}$$
$$= \frac{2\pi \times 6 \times 10^{24} \times 3 \times 10^4 \times 1.5 \times 10^{11}}{6.62 \times 10^{-34}}$$
$$= 25.61 \times 10^{73} = 2.6 \times 10^{74}$$

Hence, the quanta number that characterizes the Earth' revolution is 2.6×10^{74}.

Question 12.11:

Answer the following questions, which help you understand the difference between Thomson's model and Rutherford's model better.

Is the average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

Is the probability of backward scattering (i.e., scattering of α-particles at angles greater than 90°) predicted by Thomson's model much less, about the same, or much greater than that predicted by Rutherford's model?

Keeping other factors fixed, it is found experimentally that for small thickness t, the number of α-particles scattered at moderate angles is proportional to t. What clue does this linear dependence on t provide?

In which model is it completely wrong to ignore multiple scattering for the calculation of average angle of scattering of α-particles by a thin foil?

Answer

about the same

The average angle of deflection of α-particles by a thin gold foil predicted by Thomson's model is about the same size as predicted by Rutherford's model. This is because the average angle was taken in both models.

much less

The probability of scattering of α-particles at angles greater than 90° predicted by Thomson's model is much less than that predicted by Rutherford's model.

Scattering is mainly due to single collisions. The chances of a single collision increases linearly with the number of target atoms. Since the number of target atoms increase with an increase in thickness, the collision probability depends linearly on the thickness of the target.

Thomson's model

It is wrong to ignore multiple scattering in Thomson's model for the calculation of average angle of scattering of α–particles by a thin foil. This is because a single collision causes very little deflection in this model. Hence, the observed average scattering angle can be explained only by considering multiple scattering.

Question 12.12:

The gravitational attraction between electron and proton in a hydrogen atom is weaker than the coulomb attraction by a factor of about 10^{-40}. An alternative way of looking at this fact is to estimate the radius of the first Bohr orbit of a hydrogen atom if the electron and proton were bound by gravitational attraction. You will find the answer interesting.

Answer

Radius of the first Bohr orbit is given by the relation,

$$r_1 = \frac{4\pi \in_0 \left(\frac{h}{2\pi}\right)^2}{m_e e^2} \quad \text{... (1)}$$

Where,

$\in_0$ = Permittivity of free space

h = Planck's constant = 6.63×10^{-34} Js

m_e = Mass of an electron = 9.1×10^{-31} kg

e = Charge of an electron = 1.9×10^{-19} C

m_p = Mass of a proton = 1.67×10^{-27} kg

r = Distance between the electron and the proton

Coulomb attraction between an electron and a proton is given as:

$$F_C = \frac{e^2}{4\pi \in_0 r^2} \quad \text{... (2)}$$

Gravitational force of attraction between an electron and a proton is given as:

$$F_G = \frac{G m_p m_e}{r^2} \quad \text{... (3)}$$

Where,

G = Gravitational constant = 6.67×10^{-11} N m^2/kg^2

If the electrostatic (Coulomb) force and the gravitational force between an electron and a proton are equal, then we can write:

$\therefore F_G = F_C$

$$\frac{G m_p m_e}{r^2} = \frac{e^2}{4\pi \in_0 r^2}$$

$$\therefore \frac{e^2}{4\pi \in_0} = G m_p m_e \quad \text{... (4)}$$

Putting the value of equation (4) in equation (1), we get:

$$r_1 = \frac{\left(\frac{h}{2\pi}\right)^2}{Gm_p m_e^2}$$

$$= \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)^2}{6.67\times10^{-11}\times1.67\times10^{-27}\times\left(9.1\times10^{-31}\right)^2} \approx 1.21\times10^{29}\ m$$

It is known that the universe is 156 billion light years wide or 1.5×10^{27} m wide. Hence, we can conclude that the radius of the first Bohr orbit is much greater than the estimated size of the whole universe.

Question 12.13:

Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level *n* to level (*n*−1). For large *n*, show that this frequency equals the classical frequency of revolution of the electron in the orbit.

Answer

It is given that a hydrogen atom de-excites from an upper level (*n*) to a lower level (*n*−1).

We have the relation for energy (E_1) of radiation at level *n* as:

$$E_1 = h\nu_1 = \frac{hme^4}{(4\pi)^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3} \times \left(\frac{1}{n^2}\right) \qquad \text{... (i)}$$

Where,

ν_1 = Frequency of radiation at level n

h = Planck's constant

m = Mass of hydrogen atom

e = Charge on an electron

$\in_0$ = Permittivity of free space

Now, the relation for energy (E_2) of radiation at level ($n - 1$) is givenas:

$$E_2 = h\nu_2 = \frac{hme^4}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3} \times \frac{1}{(n-1)^2} \quad \dots \text{(ii)}$$

Where,

ν_2 = Frequency of radiation at level $(n-1)$

Energy (E) released as a result of de-excitation:

$E = E_2 - E_1$

$h\nu = E_2 - E_1$ … (iii)

Where,

ν = Frequency of radiation emitted

Putting values from equations (i) and (ii) in equation (iii), we get:

$$\nu = \frac{me^4}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3} \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{me^4 (2n-1)}{(4\pi)^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^2 (n-1)^2}$$

For large n, we can write $(2n-1) \simeq 2n$ and $(n-1) \simeq n$.

$$\therefore \nu = \frac{me^4}{32\pi^3 \epsilon_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \quad \dots \text{(iv)}$$

Classical relation of frequency of revolution of an electron is given as:

$$\nu_c = \frac{v}{2\pi r} \quad \dots \text{(v)}$$

Where,

Velocity of the electron in the n^{th} orbit is given as:

$$v = \frac{e^2}{4\pi \epsilon_0 \left(\frac{h}{2\pi}\right) n} \quad \dots \text{(vi)}$$

And, radius of the n^{th} orbit is given as:

$$r = \frac{4\pi \in_0 \left(\frac{h}{2\pi}\right)^2}{me^2} n^2 \qquad \text{... (vii)}$$

Putting the values of equations (vi) and (vii) in equation (v), we get:

$$\nu_c = \frac{me^4}{32\pi^3 \in_0^2 \left(\frac{h}{2\pi}\right)^3 n^3} \qquad \text{... (viii)}$$

Hence, the frequency of radiation emitted by the hydrogen atom is equal to its classical orbital frequency.

Question 12.14:

Classically, an electron can be in any orbit around the nucleus of an atom. Then what determines the typical atomic size? Why is an atom not, say, thousand times bigger than its typical size? The question had greatly puzzled Bohr before he arrived at his famous model of the atom that you have learnt in the text. To simulate what he might well have done before his discovery, let us play as follows with the basic constants of nature and see if we can get a quantity with the dimensions of length that is roughly equal to the known size of an atom (~ 10^{-10} m).

Construct a quantity with the dimensions of length from the fundamental constants e, m_e, and c. Determine its numerical value.

You will find that the length obtained in (a) is many orders of magnitude smaller than the atomic dimensions. Further, it involves c. But energies of atoms are mostly in non-relativistic domain where c is not expected to play any role. This is what may have suggested Bohr to discard c and look for 'something else' to get the right atomic size. Now, the Planck's constant h had already made its appearance elsewhere. Bohr's great insight lay in recognising that h, m_e, and e will yield the right atomic size. Construct a quantity with the dimension of length from h, m_e, and e and confirm that its numerical value has indeed the correct order of magnitude.

Answer

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Speed of light, $c = 3 \times 10^8$ m/s

Let us take a quantity involving the given quantities as $\left(\frac{e^2}{4\pi \in_0 m_e c^2}\right)$.

Where,

$\in_0$ = Permittivity of free space

And, $\frac{1}{4\pi \in_0} = 9\times 10^9 \text{ N m}^2 \text{ C}^{-2}$

The numerical value of the taken quantity will be:

$$\frac{1}{4\pi \in_0} \times \frac{e^2}{m_e c^2}$$
$$= 9\times 10^9 \times \frac{\left(1.6\times 10^{-19}\right)^2}{9.1\times 10^{-31} \times \left(3\times 10^8\right)^2}$$
$$= 2.81\times 10^{-15} \text{ m}$$

Hence, the numerical value of the taken quantity is much smaller than the typical size of an atom.

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Mass of an electron, $m_e = 9.1 \times 10^{-31}$ kg

Planck's constant, $h = 6.63 \times 10^{-34}$ Js

Let us take a quantity involving the given quantities as $\frac{4\pi \in_0 \left(\frac{h}{2\pi}\right)^2}{m_e e^2}$.

Where,

$\in_0$ = Permittivity of free space

And, $\frac{1}{4\pi \in_0} = 9\times 10^9 \text{ N m}^2 \text{ C}^{-2}$

The numerical value of the taken quantity will be:

$$4\pi \in_0 \times \frac{\left(\frac{h}{2\pi}\right)^2}{m_e e^2}$$

$$= \frac{1}{9\times10^9} \times \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)^2}{9.1\times10^{-31}\times\left(1.6\times10^{-19}\right)^2}$$

$$= 0.53\times10^{-10}\ m$$

Hence, the value of the quantity taken is of the order of the atomic size.

Question 12.15:

The total energy of an electron in the first excited state of the hydrogen atom is about −3.4 eV.

What is the kinetic energy of the electron in this state?

What is the potential energy of the electron in this state?

Which of the answers above would change if the choice of the zero of potential energy is changed?

Answer

Total energy of the electron, $E = -3.4$ eV

Kinetic energy of the electron is equal to the negative of the total energy.

$\Rightarrow K = -E$

$= -(-3.4) = +3.4$ eV

Hence, the kinetic energy of the electron in the given state is +3.4 eV.

Potential energy (U) of the electron is equal to the negative of twice of its kinetic energy.

$\Rightarrow U = -2\ K$

$= -2 \times 3.4 = -6.8$ eV

Hence, the potential energy of the electron in the given state is − 6.8 eV.

The potential energy of a system depends on the reference point taken. Here, the potential energy of the reference point is taken as zero. If the reference point is changed, then the value of the potential energy of the system also changes. Since total energy is the sum of kinetic and potential energies, total energy of the system will also change.

Question 12.16:

If Bohr's quantisation postulate (angular momentum = $nh/2\pi$) is a basic law of nature, it should be equally valid for the case of planetary motion also. Why then do we never speak of quantisation of orbits of planets around the sun?

Answer

We never speak of quantization of orbits of planets around the Sun because the angular momentum associated with planetary motion is largely relative to the value of Planck's constant *(h)*. The angular momentum of the Earth in its orbit is of the order of $10^{70}h$. This leads to a very high value of quantum levels *n* of the order of 10^{70}. For large values of *n*, successive energies and angular momenta are relatively very small. Hence, the quantum levels for planetary motion are considered continuous.

Question 12.17:

Obtain the first Bohr's radius and the ground state energy of a *muonic hydrogen atom* [i.e., an atom in which a negatively charged muon (μ^-) of mass about $207m_e$ orbits around a proton].

Answer

Mass of a negatively charged muon, $m_\mu = 207m_e$

According to Bohr's model,

Bohr radius, $r_e \propto \left(\frac{1}{m_e}\right)$

And, energy of a ground state *electronic hydrogen atom*, $E_e \propto m_e$.

Also, energy of a ground state *muonic hydrogen atom*, $E_\mu \propto m_\mu$.

We have the value of the first Bohr orbit, $r_e = 0.53\ \text{Å} = 0.53 \times 10^{-10}$ m

Let r_μ be the radius of *muonic hydrogen atom*.

At equilibrium, we can write the relation as:

$$
\begin{aligned}
&m_\mu r_\mu = m_e r_e \\
&207 m_e \times r_\mu = m_e r_e \\
&\therefore r_\mu = \frac{0.53 \times 10^{-10}}{207} = 2.56 \times 10^{-13}\ \text{m}
\end{aligned}
$$

Hence, the value of the first Bohr radius of a *muonic hydrogen atom* is

2.56×10^{-13} m.

We have,

$E_e = -13.6$ eV

Take the ratio of these energies as:

$$
\begin{aligned}
&\frac{E_e}{E_\mu} = \frac{m_e}{m_\mu} = \frac{m_e}{207 m_e} \\
&E_\mu = 207 E_e \\
&\quad = 207 \times (-13.6) = -2.81\ \text{keV}
\end{aligned}
$$

Hence, the ground state energy of a *muonic hydrogen atom* is −2.81 keV.