

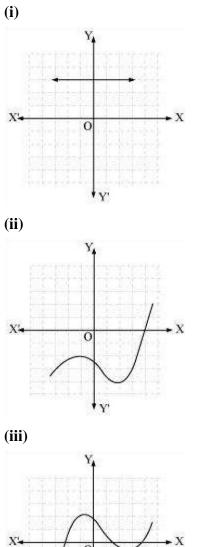
www.eduinput.com NCERT Solutions for Class 10 Maths Unit 2

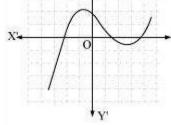
Polynomials Class 10

Unit 2 Polynomials Exercise 2.1, 2.2, 2.3 2.4, 2.4 Solutions

Exercise 2.1 : Solutions of Questions on Page Number : 28 Q1 :

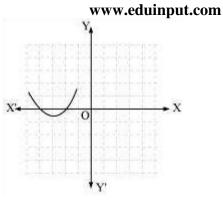
The graphs of y = p(x) are given in following figure, for some polynomials p(x). Find the number of zeroes of p(x), in each case.



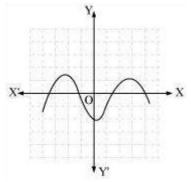




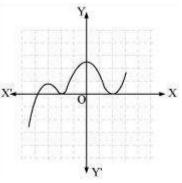












Answer :

- (i) The number of zeroes is 0 as the graph does not cut the *x*-axis at any point.
- (ii) The number of zeroes is 1 as the graph intersects the *x*-axis at only 1 point.
- (iii) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.
- (iv) The number of zeroes is 2 as the graph intersects the *x*-axis at 2 points. (v) The number of zeroes is 4 as the graph intersects the *x*-axis at 4 points.
- (vi) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.



Exercise 2.2 : Solutions of Questions on Page Number : 33 Q1 :

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i)
$$x^{2} - 2x - 8$$
 (ii) $4s^{2} - 4s + 1$ (iii) $6x^{2} - 3 - 7x$
(iv) $4u^{2} + 8u$ (v) $t^{2} - 15$ (vi) $3x^{2} - x - 4$

Answer :

(i) $x^2 - 2x - 8 = (x - 4)(x + 2)$

The value of $x^2 - 2x - 8$ is zero when x - 4 = 0 or x + 2 = 0, i.e., when x = 4 or x = -2Therefore, the zeroes of $x^2 - 2x - 8$ are 4 and -2.

$$= 4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of }x)}{\text{Coefficient of }x^2}$$

Sum of zeroes =

$$4 \times (-2) = -8 = \frac{(-8)}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

Product of zeroes

(ii)
$$4s^2 - 4s + 1 = (2s - 1)^2$$

The value of $4s^2 - 4s + 1$ is zero when 2s - 1 = 0, i.e., $s = \frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$.

Sum of zeroes =
$$\frac{\frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$$

Product of zeroes
$$=\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{Constant term}}{\text{Coefficient of } s^2}$$

(iii)
$$6x^2 - 3 - 7x = 6x^2 - 7x - 3 = (3x + 1)(2x - 3)$$

 $x = \frac{-1}{3}$ or $x = \frac{3}{2}$

The value of $6x^2 - 3 - 7x$ is zero when 3x + 1 = 0 or 2x - 3 = 0, i.e.,



Therefore, the zeroes of
$$6x^2 - 3 - 7x$$
 are $\frac{-1}{3}$ and $\frac{3}{2}$.
Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$
Product of zeroes = $\frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$
(iv) $4u^2 + 8u = 4u^2 + 8u + 0$
 $= 4u(u+2)$

The value of $4u^2 + 8u$ is zero when 4u = 0 or u + 2 = 0, i.e., u = 0 or u = -2Therefore, the zeroes of $4u^2 + 8u$ are 0 and - 2.

Sum of zeroes = $0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes = $0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(v)
$$t^2 - 15$$

= $t^2 - 0t - 15$
= $(t - \sqrt{15})(t + \sqrt{15})$

The value of t^2 - 15 is zero when $t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$, i.e., when

Q2 :

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i)
$$\frac{1}{4}$$
, -1 (ii) $\sqrt{2}$, $\frac{1}{3}$ (iii) 0 , $\sqrt{5}$
(iv) 1,1 (v) $-\frac{1}{4}$, $\frac{1}{4}$ (vi) 4,1

Answer :

(i) $\frac{1}{4}, -1$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α β and



$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$
$$\alpha \beta = -1 = \frac{-4}{4} = \frac{c}{a}$$
If $a = 4$, then $b = -1$, $c = -4$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii)
$$\sqrt{2}, \frac{1}{3}$$

Let the polynomial be $ax^2 + bx + c$

β

.

α

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$

$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$

If $a = 3$, then $b = -3\sqrt{2}$, $c = 1$, and its zeroes be and

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0,\sqrt{5}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes α β_d

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$
$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$
If $a = 1$, then $b = 0$, $c = \sqrt{5}$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

Let the polynomial be $ax^2 + bx + c$, and its zeroes α β_d

$$\alpha + \beta = 1 = \frac{1}{1} = \frac{-b}{a}$$
$$\alpha \times \beta = 1 = \frac{1}{1} = \frac{c}{a}$$
If $a = 1$, then $b = -1$, $c = 1$



Therefore, the quadratic polynomial is $x^2 - x + 1$.

$$(v) \quad -\frac{1}{4}, \frac{1}{4}$$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be α and

Exercise 2.3 2.4 : Solutions of Questions on Page Number : 36

Q1 :

Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3,$ $g(x) = x^2 - 2$ (ii) $p(x) = x^4 - 3x^2 + 4x + 5,$ $g(x) = x^2 + 1 - x$ (iii) $p(x) = x^4 - 5x + 6,$ $g(x) = 2 - x^2$

Answer :

(i)
$$p(x) = x^3 - 3x^2 + 5x - 3$$

 $q(x) = x^2 - 2$
 $x^2 - 2$
 $x^3 - 3x^2 + 5x - 3$
 $x^3 - 2x$
 $- +$
 $-3x^2 + 7x - 3$
 $-3x^2 + 6$
 $+ -$
 $7x - 9$

Quotient = x - 3Remainder = 7x - 9

(ii)
$$p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0 \cdot x^3 - 3x^2 + 4x + 5$$

 $q(x) = x^2 + 1 - x = x^2 - x + 1$

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$$\begin{array}{r} x^{2} + x - 3 \\ x^{2} - x + 1 \hline x^{4} + 0 \cdot x^{3} - 3x^{2} + 4x + 5 \\ x^{4} - x^{3} + x^{2} \\ - + - \\ \hline x^{3} - 4x^{2} + 4x + 5 \\ x^{3} - x^{2} + x \\ - + - \\ \hline - 3x^{2} + 3x + 5 \\ - 3x^{2} + 3x - 3 \\ \hline + - + \\ \hline 8 \\ \end{array}$$

Quotient = $x^2 + x - 3$

Remainder = 8

(iii)
$$p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$$

 $q(x) = 2 - x^2 = -x^2 + 2$
 $-x^2 + 2\overline{\smash{\big)}} \frac{-x^2 - 2}{x^4 + 0.x^2 - 5x + 6}$
 $x^4 - 2x^2$
 $- +$
 $2x^2 - 5x + 6$
 $2x^2 - 4$
 $- +$
 $-5x + 10$

Quotient = $-x^2 - 2$ Remainder = -5x + 10

Q2 :

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i)
$$2x^3 + x^2 - 5x + 2;$$
 $\frac{1}{2}, 1, -2$
(ii) $x^3 - 4x^2 + 5x - 2;$ 2,1,1



Answer :

(i) $p(x) = 2x^3 + x^2 - 5x + 2.$ Zeroes for this polynomial are $\frac{1}{2}, 1, -2$ $p(\frac{1}{2}) = 2(\frac{1}{2})^3 + (\frac{1}{2})^2 - 5(\frac{1}{2}) + 2$

$$= \frac{1}{4} + \frac{1}{4} - \frac{3}{2} + 2$$

= 0
$$p(1) = 2 \times 1^{3} + 1^{2} - 5 \times 1 + 2$$

= 0
$$p(-2) = 2(-2)^{3} + (-2)^{2} - 5(-2) + 2$$

= -16 + 4 + 10 + 2 = 0
$$\frac{1}{2}$$

Therefore, 2, 1, and - 2 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain a = 2, b = 1, c = -5, d = 2

We can take
$$\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$$

 $\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$
 $\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$
 $\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii)
$$p(x) = x^3 - 4x^2 + 5x - 2$$

Zeroes for this polynomial are 2, 1, 1.

$$p(2) = 2^{3} - 4(2^{2}) + 5(2) - 2$$

= 8 - 16 + 10 - 2 = 0
$$p(1) = 1^{3} - 4(1)^{2} + 5(1) - 2$$

= 1 - 4 + 5 - 2 = 0

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + di = 1$, b = -4, c = 5, d = -2. Verification of the relationship between zeroes and coefficient of the given polynomial

Sum of zeroes =
$$2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$$

Multiplication of zeroes taking two at a time = $(2)(1) + (1)(1) + (2)(1) = 2 + 1 + 2 = 5 = \frac{(5)}{1} = \frac{c}{a}$

$$=\frac{-(-2)}{1}=\frac{-d}{a}$$

Multiplication of zeroes = $2 \times 1 \times 1 = 2$ 1

Hence, the relationship between the zeroes and the coefficients is verified.

Q3 :

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i)
$$t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$
(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Answer :

(i)
$$t^2 - 3$$
, $2t^4 + 3t^3 - 2t^2 - 9t - 12$
 $t^2 - 3 = t^2 + 0.t - 3$
 $t^2 + 0.t - 3$) $2t^2 + 3t + 4$
 $t^2 + 0.t - 3$) $2t^4 + 3t^3 - 2t^2 - 9t - 12$
 $2t^4 + 0.t^3 - 6t^2$
 $- - +$
 $3t^3 + 4t^2 - 9t - 12$
 $3t^3 + 0.t^2 - 9t$
 $- - +$
 $4t^2 + 0.t - 12$
 $4t^2 + 0.t - 12$
 $- - +$
 0

Since the remainder is 0,

Hence,
$$t^2 - 3$$
 is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.
(ii) $x^2 + 3x + 1$, $3x^4 + 5x^3 - 7x^2 + 2x + 2$
 $x^2 + 3x + 1$) $3x^4 + 5x^3 - 7x^2 + 2x + 2$
 $3x^4 + 9x^3 + 3x^2$
 $- - -$
 $-4x^3 - 10x^2 + 2x + 2$
 $-4x^3 - 12x^2 - 4x$
 $+ + +$
 $2x^2 + 6x + 2$
 0

Since the remainder is 0,

Hence, $x^{2} + 3x + 1$ is a factor of $3x^{4} + 5x^{3} - 7x^{2} + 2x + 2$. (iii) $x^{3} - 3x + 1$, $x^{5} - 4x^{3} + x^{2} + 3x + 1$ $x^{3} - 3x + 1) \overline{x^{5} - 4x^{3} + x^{2} + 3x + 1}$ $x^{5} - 3x^{3} + x^{2}$ - + - $-x^{3} + 3x + 1$ $-x^{3} + 3x - 1$ + - - + 2

Since the remainder $\neq 0$,

Hence, $x^3 - 3x + 1$ is not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

Q4 :

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, -7, -14 respectively.

Answer :

Let the polynomial be $ax^3 + bx^2 + cx + d$

It is given that

$$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$$
$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$
$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If a = 1, then b = -2, c = -7, d = 14

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$. and the zeroes be α, β , and γ .

Q5 :

Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer:

$$3x^{4} + 6x^{3} - 2x^{2} - 10x - 5$$

$$p(x) = 3x^{4} + 6x^{3} - 2x^{2} - 10x - 5$$

$$x^{2} - \frac{5}{3}$$
Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$,
 $\therefore \left(x - \sqrt{\frac{5}{3}}\right) \left(x + \sqrt{\frac{5}{3}}\right) = \left(x^{2} - \frac{5}{3}\right)$
is a factor of .

Therefore, we divide the given polynomial by

$$x^{2} + 0.x - \frac{5}{3} \frac{3x^{2} + 6x + 3}{3x^{4} + 6x^{3} - 2x^{2} - 10x - 5}}{3x^{4} + 0x^{3} - 5x^{2}}$$

$$- - + \frac{6x^{3} + 3x^{2} - 10x - 5}{6x^{3} + 0x^{2} - 10x}$$

$$- - + \frac{3x^{2} + 0x - 5}{3x^{2} + 0x - 5}$$

$$- - - + \frac{0}{3x^{4} + 6x^{3} - 2x^{2} - 10x - 5} = \left(x^{2} - \frac{5}{3}\right)\left(3x^{2} + 6x + 3\right)$$

$$= 3\left(x^{2} - \frac{5}{3}\right)\left(x^{2} + 2x + 1\right)$$

We factorize $x^2 + 2x + 1$ = $(x+1)^2$

Therefore, its zero is given by x + 1 = 0

x = -1

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at x = -1.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}$, $-\sqrt{\frac{5}{3}}$, -1 and -1.

Q6 :

On dividing $x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were x - 2 and -2x + 4, respectively. Find g(x).

Answer :

$$p(x) = x^3 - 3x^2 + x + 2 \qquad \text{(Dividend)}$$

g(x) = ? (Divisor) Quotient = (x - 2) Remainder = (-2x + 4)

 $Dividend = Divisor \times Quotient + Remainder$

$$x^{3} - 3x^{2} + x + 2 = g(x) \times (x - 2) + (-2x + 4)$$

$$x^{3} - 3x^{2} + x + 2 + 2x - 4 = g(x)(x - 2)$$

$$x^{3} - 3x^{2} + 3x - 2 = g(x)(x - 2)$$

g(x) is the quotient when we divide $\begin{pmatrix} x^3 - 3x^2 + 3x - 2 \end{pmatrix}$ by $\begin{pmatrix} x - 2 \end{pmatrix}$

$$\begin{array}{r} x-2) \overline{x^{2}-x+1} \\ x-2) \overline{x^{3}-3x^{2}+3x-2} \\ x^{3}-2x^{2} \\ -+ \\ -x^{2}+3x-2 \\ -x^{2}+2x \\ +- \\ x-2 \\ x-2 \\ -- + \\ \hline 0 \\ \hline \\ \therefore g(x) = (x^{2}-x+1) \end{array}$$

Q7 :

Give examples of polynomial p(x), g(x), q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x)

(ii)
$$\deg q(x) = \deg r(x)$$

(iii) deg r(x) = 0

Answer :

According to the division algorithm, if p(x) and g(x) are two polynomials with

 $g(x) \neq 0$, then we can find polynomials q(x) and r(x) such that $p(x) = g(x) \times p(x)$

q(x) + r(x), where r(x) = 0 or degree of $r(x) \le degree$ of g(x)

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg p(x) = deg q(x)

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by 2. Here, $p(x) = 6x^2 + 2x + 2$ g(x) = 2 $q(x) = 3x^2 + x + 1$ and r(x) = 0Degree of p(x) and q(x) is the same i.e., 2. Checking for division algorithm, p(x) $= g(x) \times q(x) + r(x)$ $6x^2 + 2x + 2 = 2(3x^2 + x + 1)$ $-6x^{2}+2x+2$) Thus, the division algorithm is satisfied. (ii) deg $q(x) = \deg r(x)$ Let us assume the division of $x^3 + x$ by x^2 , Here, $p(x) = x^3 + x g(x) = x^2 q(x) = x$ and r(x) = xClearly, the degree of q(x) and r(x) is the same i.e., 1. Checking for division algorithm, $p(x) = g(x) \times q(x) +$ $r(x) x^3 + x = (x^2) \times x + x x^3 + x = x^3 + x$ Thus, the division algorithm is satisfied.

(iii)deg r(x) = 0

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of x^3 + 1by x^2 .

Here, $p(x) = x^3 + 1$ $g(x) = x^2 q(x) = x$ and

r(x) = 1

Clearly, the degree of r(x) is 0.

Checking for division algorithm,

 $p(x) = g(x) \times q(x) + r(x) x^3 + 1 =$

 $(x^2) \times x + 1 x^3 + 1 = x^3 + 1$

Exercise 2.4 : Solutions of Questions on Page Number : 37

Q1 :

Thus, the division algorithm is satisfied.

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are a - b, a, a + b, find a and b.

Answer :

 $p(x) = x^3 - 3x^2 + x + 1$ Zeroes are a - b, a + a + bComparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain p = 1, q = -3, r = 1, t = 1Sum of zeroes = a - b + a + a + b $\frac{-q}{p} = 3a$ $\frac{-(-3)}{1} = 3a$ 3 = 3aa = 1The zeroes are 1-b, 1, 1+b. Multiplication of zeroes = 1(1-b)(1+b) $\frac{-t}{p} = 1 - b^2$ $\frac{-1}{1} = 1 - b^2$ $1-b^2 = -1$ $1+1=b^2$ $b = \pm \sqrt{2}$

Hence, a = 1 and $b = \sqrt{2}$ or $-\sqrt{2}$.

Q2 :

]It two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Answer :

Given

$$\sqrt{3}$$
 and $2 - \sqrt{3}$ that 2 + are zeroes of the given polynomial.
 $(x-2-\sqrt{3})(x-2+\sqrt{3})$ Therefore, $= x^2 + 4 - 4x - 3$

 $= x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$\begin{array}{r} x^2 - 2x - 35 \\ x^2 - 4x + 1 \overline{\smash{\big)}} x^4 - 6x^3 - 26x^2 + 138x - 35 \\ x^4 - 4x^3 + x^2 \\ - 4x^3 + x^3 + x^2 \\ - 4x^3 + x^3 + x^3 \\ - 4x^3 + x^3 + x^3 \\ - 4x^3$$

Clearly, $x^4 - 6x^3 - 26x^2 + 138x - 35 = (x^2 - 4x + 1)(x^2 - 2x - 35)$

It can be observed that $(x^2 - 2x - 35)$ is also a factor of the given polynomial.

And
$$\binom{x^2 - 2x - 35}{x - 35} = \binom{x - 7}{x + 5}$$

Therefore, the value of the polynomial is also zero when x-7=0 or x+5=0

Or x = 7 or - 5

Hence, 7 and - 5 are also zeroes of this polynomial.

Q3 :

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be x + a, find k and a.

Answer :

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend - Remainder = Divisor × Quotient

$$x^{4} - 6x^{3} + 16x^{2} - 25x + 10 - x - a = x^{4} - 6x^{3} + 16x^{2} - 26x + 10 - a$$
 will be perfectly divisible
by $x^{2} - 2x + k$.

Let us divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$

- 5 - *a* = 0 Therefore, *a* = - 5 Hence, *k* = 5 and *a* = - 5