

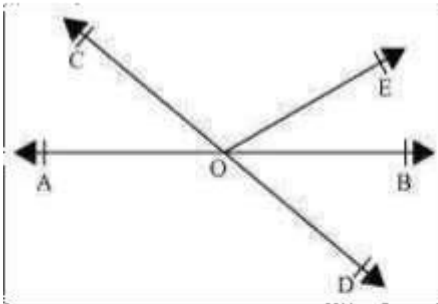
Class IX Chapter 6 – Lines and Angles Maths

Exercise 6.1 Question 1:

In the given figure, lines AB and CD intersect at O. If $\angle AOC + \angle BOE = 70^\circ$ and $a = 2x$, and $b = 3x$

$\angle BOD = 40^\circ$,

find $\angle BOE$ and reflex $\angle COE$.



Answer:

AB is a straight line, rays OC and OE stand on it.

$$\therefore \angle AOC + \angle COE + \angle BOE = 180^\circ$$

$$\Rightarrow (\angle AOC + \angle BOE) + \angle COE = 180^\circ$$

$$\Rightarrow 70^\circ + \angle COE = 180^\circ$$

$$\Rightarrow \angle COE = 180^\circ - 70^\circ = 110^\circ$$

$$\text{Reflex } \angle COE = 360^\circ - 110^\circ = 250^\circ$$

CD is a straight line, rays OE and OB stand on it.

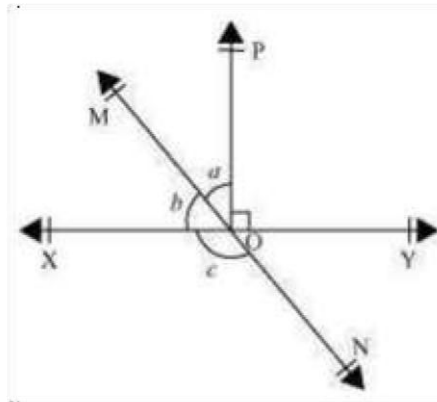
$$\therefore \angle COE + \angle BOE + \angle BOD = 180^\circ$$

$$\Rightarrow 110^\circ + \angle BOE + 40^\circ = 180^\circ$$

$$\Rightarrow \angle BOE = 180^\circ - 150^\circ = 30^\circ$$

Question 2:

In the given figure, lines XY and MN intersect at O. If $\angle POY = 90^\circ$ and $a:b = 2 : 3$, find c.



Answer:

Let the common ratio between a and b be x. \therefore
 XY is a straight line, rays OM and OP stand on it.

$$\therefore \angle XOM + \angle MOP + \angle POY = 180^\circ \quad b + a + \angle POY = 180^\circ$$

$$3x + 2x + 90^\circ = 180^\circ \quad 5x = 90^\circ \quad x = 18^\circ \quad a =$$

$$2x = 2 \times 18 = 36^\circ \quad b =$$

$$3x = 3 \times 18 = 54^\circ$$

MN is a straight line. Ray OX stands on it.

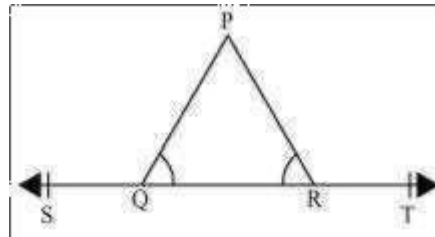
$$\therefore b + c = 180^\circ \text{ (Linear Pair)}$$

$$54^\circ + c = 180^\circ \quad c = 180^\circ -$$

$$54^\circ = 126^\circ \quad \therefore c = 126^\circ$$

Question 3:

In the given figure, $\angle PQR = \angle PRQ$, then prove that $\angle PQS = \angle PRT$.



Answer:

In the given figure, ST is a straight line and ray QP stands on it.

$$\therefore \angle PQS + \angle PQR = 180^\circ \text{ (Linear Pair)}$$

$$\angle PQR = 180^\circ - \angle PQS \text{ (1)}$$

$$\angle PRT + \angle PRQ = 180^\circ \text{ (Linear Pair)}$$

$$\angle PRQ = 180^\circ - \angle PRT \text{ (2)}$$

It is given that $\angle PQR = \angle PRQ$.

Equating equations (1) and (2), we obtain

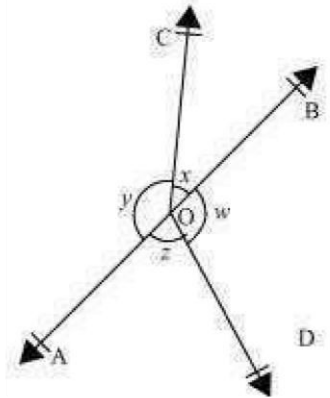
$$180^\circ - \angle PQS = 180^\circ - \angle PRT$$

$$= \angle PRT$$

Question 4:

$$x + y = w + z,$$

In the given figure, if



Answer:

It can be observed that, $x + y + z + w$ then prove that AOB is a line.

$= 360^\circ$ (Complete angle) It is given that, $x + y = z + w \therefore x + y + x + y$

$$= 360^\circ$$

$$2(x + y) = 360^\circ$$

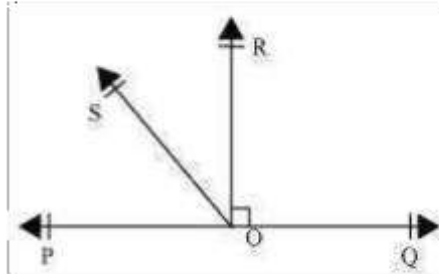
$$x + y = 180^\circ$$

Since x and y form a linear pair, AOB is a line.

Question 5:

In the given figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that

$$\angle ROS = \frac{1}{2}(\angle QOS - \angle POS).$$



Answer:

It is given that $OR \perp PQ$

$$\therefore \angle POR = 90^\circ$$

$$\therefore \angle POS + \angle SOR = 90^\circ$$

$$\therefore \angle ROS = 90^\circ - \angle POS \dots (1)$$

$$\therefore \angle QOR = 90^\circ \text{ (As } OR \perp PQ)$$

$$\therefore \angle QOS - \angle ROS = 90^\circ$$

$$\therefore \angle ROS = \angle QOS - 90^\circ \dots (2)$$

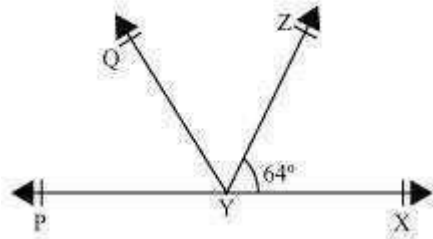
On adding equations (1) and (2), we obtain

$$\begin{aligned} \therefore \angle ROS &= \angle QOS - 90^\circ \\ \angle ROS &= \frac{1}{2}(\angle QOS - \angle POS) \end{aligned}$$

Question 6:

It is given that $\angle XYZ = 64^\circ$ and XY is produced to point P. Draw a figure from the given information. If ray YQ bisects $\angle ZYP$, find $\angle XYQ$ and reflex $\angle QYP$.

Answer:



It is given that line YQ bisects \angle PYZ.

Hence, \angle QYP = \angle ZYQ

It can be observed that PX is a line. Rays YQ and YZ stand on it.

$$\angle$$
 XYZ + \angle ZYQ + \angle QYP = 180°

$$64^\circ + 2 \angle$$
 QYP = 180°

$$2 \angle$$
 QYP = $180^\circ - 64^\circ = 116^\circ$

$$\angle$$
 QYP = 58°

Also, \angle ZYQ = \angle QYP = 58°

Reflex \angle QYP = $360^\circ - 58^\circ = 302^\circ$

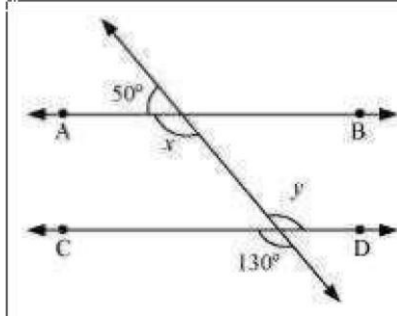
$$\angle$$
 XYQ = \angle XYZ + \angle ZYQ

$$= 64^\circ + 58^\circ = 122^\circ$$

Exercise 6.2 Question

1:

In the given figure, find the values of x and y and then show that $AB \parallel CD$.



Answer:

It can be observed that, 50°

$+ x = 180^\circ$ (Linear pair) $x =$

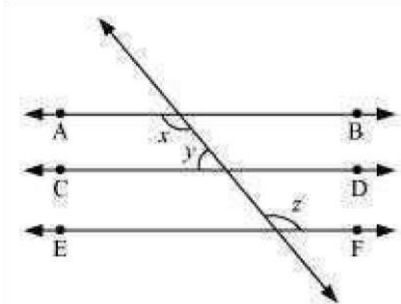
$130^\circ \dots (1)$

Also, $y = 130^\circ$ (Vertically opposite angles)

As x and y are alternate interior angles for lines AB and CD and also measures of these angles are equal to each other, therefore, line $AB \parallel CD$.

Question 2:

In the given figure, if $AB \parallel CD$, $CD \parallel EF$ and $y : z = 3 : 7$, find x .



Answer:

It is given that $AB \parallel CD$ and $CD \parallel EF$

$\therefore AB \parallel CD \parallel EF$ (Lines parallel to the same line are parallel to each other)

It can be observed that $x = z$

(Alternate interior angles) ... (1)

It is given that $y : z = 3 : 7$

Let the common ratio between y and z be a . \therefore

$y = 3a$ and $z = 7a$

Also, $x + y = 180^\circ$ (Co-interior angles on the same side of the transversal) z

$+ y = 180^\circ$ [Using equation (1)]

$$7a + 3a = 180^\circ$$

$$10a = 180^\circ \quad a =$$

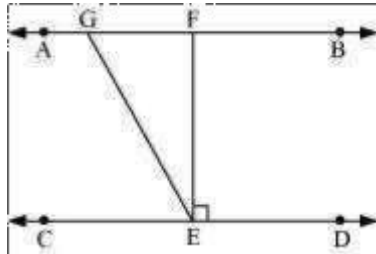
$$18^\circ \quad \therefore x = 7a = 7 \times 18^\circ =$$

$$126^\circ$$

Question 3:

In the given figure, If $AB \parallel CD$, $EF \perp CD$ and $\angle GED = 126^\circ$, find $\angle AGE$, $\angle GEF$ and $\angle =$

\angle FGE.



Answer:

It is given that,
 $AB \parallel CD$

$\therefore EF \perp CD$

$\therefore \angle GED = 126^\circ$

$\therefore \angle GEF + \angle FED = 126^\circ$

$\therefore \angle GEF + 90^\circ = 126^\circ$

$\therefore \angle GEF = 36^\circ$

$\therefore \angle AGE$ and $\angle GED$ are alternate interior angles.

$\therefore \angle AGE = \angle GED = 126^\circ$

\therefore However, $\angle AGE + \angle FGE = 180^\circ$ (Linear pair)

$\therefore 126^\circ + \angle FGE = 180^\circ$

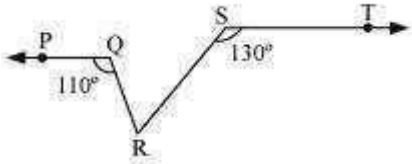
$\angle FGE = 180^\circ - 126^\circ = 54^\circ$

$\angle AGE = 126^\circ, \angle GEF = 36^\circ, \angle FGE = 54^\circ$

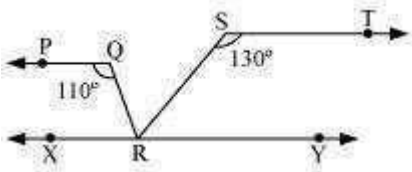
Question 4:

In the given figure, if $PQ \parallel ST$, $\angle PQR = 110^\circ$ and $\angle RST = 130^\circ$, find $\angle QRS$.

[Hint: Draw a line parallel to ST through point R.]



Answer:



Let us draw a line XY parallel to ST and passing through point R.

$\therefore \angle PQR + \angle QRX = 180^\circ$ (Co-interior angles on the same side of transversal QR)

$$\therefore 110^\circ + \angle QRX = 180^\circ$$

$\therefore \therefore$

$$\angle QRX = 70^\circ$$

Also,

$\therefore \angle RST + \angle SRY = 180^\circ$ (Co-interior angles on the same side of transversal SR)

$$\angle X + \angle SRY = 180^\circ - 130^\circ$$

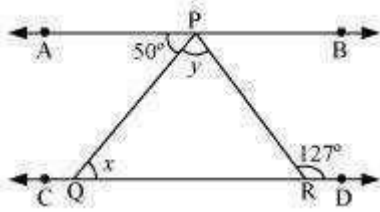
$$\therefore \angle SRY = 50^\circ$$

XY is a straight line. RQ and RS stand on it.

$$\therefore \angle QRX + \angle QRS + \angle SRY = 180^\circ \quad \angle X + \angle QRS + 50^\circ = 180^\circ - 70^\circ$$

$$\therefore \angle QRS = 180^\circ - 120^\circ = 60^\circ$$

Question 5:



Answer:

$\angle APR = \angle PRD$ (Alternate interior angles) In the given figure, if $AB \parallel CD$, $\angle APQ = 50^\circ$ and $\angle PRD = 127^\circ$, find x and y .

$$50^\circ + y = 127^\circ \quad y =$$

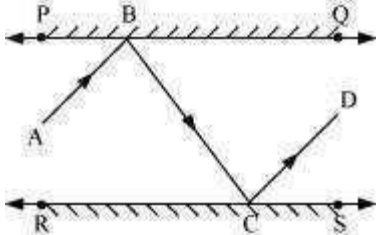
$$127^\circ - 50^\circ \quad y =$$

$$77^\circ$$

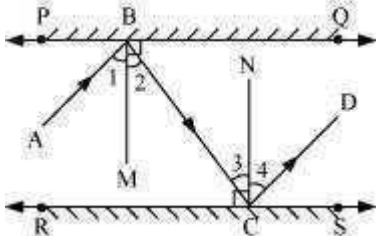
Also, $\angle APQ = \angle PQR$ (Alternate interior angles)

$$50^\circ = x \quad x = 50^\circ \text{ and } y = 77^\circ \text{ Question 6:}$$

In the given figure, PQ and RS are two mirrors placed parallel to each other. An incident ray AB strikes the mirror PQ at B , the reflected ray moves along the path BC and strikes the mirror RS at C and again reflects back along CD . Prove that $AB \parallel CD$.



Answer:



Let us draw $BM \perp PQ$ and $CN \perp RS$.

As $PQ \parallel RS$,

Therefore, $BM \parallel CN$

Thus, BM and CN are two parallel lines and a transversal line BC cuts them at B and C respectively.

$\hat{1} = \hat{3}$ (Alternate interior angles) $\hat{2}$

However, $\hat{1} = \hat{2}$ and $\hat{3} = \hat{4}$ (By laws of reflection)

$$\hat{1} = \hat{2} = \hat{3} = \hat{4}$$

Also, $\hat{1} + \hat{2} = \hat{3} + \hat{4}$

$$ABC = DCB$$

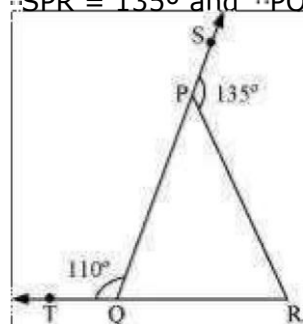
However, these are alternate interior angles. \therefore

$AB \parallel CD$

Exercise 6.3 Question

1:

In the given figure, sides QP and RQ of $\triangle PQR$ are produced to points S and T respectively. If $\angle SPR = 135^\circ$ and $\angle PQT = 110^\circ$, find $\angle PRQ$.



Answer:

∴

It is given that,

$$\angle SPR = 135^\circ \text{ and } \angle PQT = 110^\circ$$

∴

$$\angle SPR + \angle QPR = 180^\circ \text{ (Linear pair angles)}$$

∴

$$135^\circ + \angle QPR = 180^\circ$$

∴

$$\angle QPR = 45^\circ$$

Also, $\angle PQT + \angle PQR = 180^\circ$ (Linear pair angles)

∴

$$110^\circ + \angle PQR = 180^\circ$$

∴

$$\angle PQR = 70^\circ$$

As the sum of all interior angles of a triangle is 180° , therefore, for $\triangle PQR$,

$$\angle QPR + \angle PQR + \angle PRQ = 180^\circ$$

∴

$$45^\circ + 70^\circ + \angle PRQ = 180^\circ$$

∴

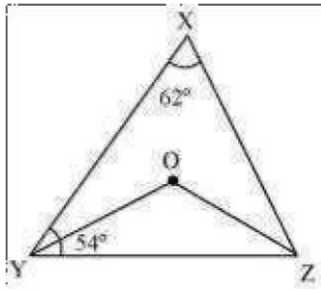
$$\angle PRQ = 180^\circ - 115^\circ$$

∴

$$\angle PRQ = 65^\circ$$

Question 2:

In the given figure, $\angle X = 62^\circ$, $\angle XYZ = 54^\circ$. If YO and ZO are the bisectors of $\angle XYZ$ and $\angle XZY$ respectively of $\triangle XYZ$, find $\angle OZY$ and $\angle YQZ$.



Answer:

As the sum of all interior angles of a triangle is 180° , therefore, for $\triangle XYZ$,

$$\angle X + \angle XYZ + \angle XZY = 180^\circ$$

$$62^\circ + 54^\circ + \angle XZY = 180^\circ$$

$$\angle XZY = 180^\circ - 116^\circ$$

$$\angle XZY = \frac{64}{2} = 32^\circ$$

$$\angle ZOY = \frac{64}{2} = 32^\circ \text{ (OZ is the angle bisector of } \angle XZY \text{)}$$

$$\angle YOZ = \frac{54}{2} = 27^\circ \text{ Similarly, } \angle YOZ = 27^\circ$$

Using angle sum property for $\triangle YOZ$, we obtain

$$\angle YOZ + \angle ZOY + \angle YOZ = 180^\circ$$

$$27^\circ + 32^\circ + \angle YOZ = 180^\circ$$

\therefore

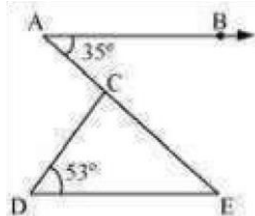
$$\angle YOZ = 180^\circ - 59^\circ$$

\therefore

$$\angle YOZ = 121^\circ$$

Question 3:

In the given figure, if $AB \parallel DE$, $\angle BAC = 35^\circ$ and $\angle CDE = 53^\circ$, find $\angle DCE$.



Answer:

$AB \parallel DE$ and AE is a transversal.

$$\angle BAC = \angle CED \text{ (Alternate interior angles)}$$

$$\therefore \angle CED = 35^\circ$$

In $\triangle CDE$,

$$\angle CDE + \angle CED + \angle DCE = 180^\circ \text{ (Angle sum property of a triangle)}$$

$$90^\circ + 35^\circ + \angle DCE = 180^\circ$$

\therefore

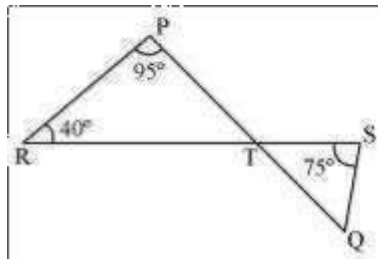
$$\angle DCE = 180^\circ - 88^\circ$$

$$\angle DCE = 92^\circ$$

Question 4:

In the given figure, if lines PQ and RS intersect at point T, such that $\angle PRT = 40^\circ$,

$\angle RPT = 95^\circ$ and $\angle TSQ = 75^\circ$, find $\angle SQT$.



Answer:

Using angle sum property for $\triangle PRT$, we obtain

$$\angle PRT + \angle RPT + \angle PTR = 180^\circ$$

$$40^\circ + 95^\circ + \angle PTR = 180^\circ$$

$$\angle PTR = 180^\circ - 135^\circ$$

$$\angle PTR = 45^\circ$$

\therefore

$$\angle STQ = \angle PTR = 45^\circ \text{ (Vertically opposite angles)}$$

\therefore

$$\angle STQ = 45^\circ$$

By using angle sum property for $\triangle STQ$, we obtain

$$\angle STQ + \angle SQT + \angle QST = 180^\circ$$

$$45^\circ + \angle SQT + 75^\circ = 180^\circ$$

\therefore

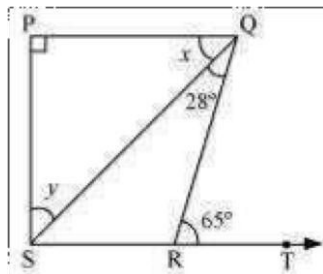
$$\angle SQT = 180^\circ - 120^\circ$$

\therefore

$$\angle SQT = 60^\circ$$

Question 5:

In the given figure, if $PQ \perp PS$, $PQ \parallel SR$, $\angle SQR = 28^\circ$ and $\angle QRT = 65^\circ$, then find the values of x and y .



Answer:

It is given that $PQ \parallel SR$ and QR is a transversal line.

$\angle PQR = \angle QRT$ (Alternate interior angles) x

$$+ 28^\circ = 65^\circ \quad x = 65^\circ - 28^\circ \quad x = 37^\circ$$

By using the angle sum property for $\triangle SPQ$, we obtain

$$\angle SPQ + x + y = 180^\circ$$

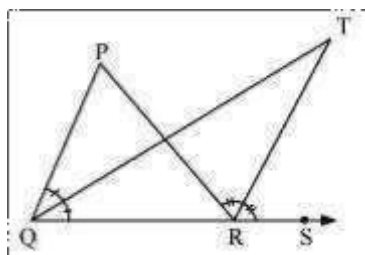
$$90^\circ + 37^\circ + y = 180^\circ \quad y$$

$$= 180^\circ - 127^\circ \quad y = 53^\circ \quad \therefore x = 37^\circ$$

and $y = 53^\circ$ Question 6:

In the given figure, the side QR of $\triangle PQR$ is produced to a point S . If the bisectors of $\angle PQR$

and $\angle PRS$ meet at point T , then prove that $\angle QTR = \frac{1}{2} \angle QPR$.



Answer:

In $\triangle QTR$, $\angle TRS$ is an exterior angle.

$$\angle QTR + \angle TQR = \angle TRS$$

$$\angle QTR = \angle TRS - \angle TQR \quad (1)$$

\therefore

\therefore

\therefore

For ΔPQR , $\angle PRS$ is an external angle.

$$\therefore \angle QPR + \angle PQR = \angle PRS$$

$$\angle QPR + 2 \angle TQR = 2 \angle TRS \text{ (As QT and RT are angle bisectors)}$$

$$\angle QPR = 2(\angle TRS - \angle TQR)$$

$$\angle QPR = 2 \angle QTR \text{ [By using equation (1)]}$$

∴